In order to see that the values of the density $\mu(t)$ given by the last formulas are actually the solutions of the singular Eqs.(4.12), the direct substitution of the densities (4.13) into the appropriate initial equations should be performed and satisfaction of the relationships

$$\frac{1}{\pi i}\int_0^\infty M^{\mp}(t, t_0)\, dt\, \frac{1}{\pi i}\int_0^\infty f(t_1)\, M^{\pm}(t_1, t)\, dt_1 = f(t_0)$$

should actually be verified.

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ON AN INTEGRAL EQUATION OF THE PROBLEM FOR AN ELASTIC STRIP WITH A SLIT*

A.V. BOIKO and L.N. KARPENKO

A new singular integral equation is obtained that describes the elastic equilibrium of a strip with both an inner and an edge slit (crack) and has a considerable advantage over existing equations /1-9/, etc.) from the viewpoint of a numerical realization and clarification of the analytical relationship with an analogous equation for a half-plane. Numerical results are given of a computation of the stress intensity coefficients at the tips of the inner and edge cracks that refine data in the literature.

1. Let an elastic body occupy the strip $0 < y < H$, $-\infty < x < \infty$ with a rectilinear slit along the Oy axis between the points $y = a$, $y = b$, $a \geqslant 0$, $b \leqslant H$. The strip boundary is stress-free, while the stresses $\sigma_x = p(y)$, $\tau_{xy} = 0$ are given on the slit edges. Then the state of stress of the body under consideration is described /10/ by using two regular functions of the complex variable $z = x + iy$:

$$\sigma_x + \sigma_y = 2\,[\Phi(z) + \overline{\Phi(z)}],\quad \sigma_y - \sigma_x + 2i\tau_{xy} = 2\,[\bar{z}\Phi'(z) + \Psi(z)] \tag{1.1}$$

that satisfy the boundary conditions on the slit edges

$$\Phi^{\pm}(iy) + \overline{\Phi^{\pm}(iy)} + iy\Phi'^{\pm}(iy) - \Psi^{\pm}(iy) = p(y) \tag{1.2}$$

and on the strip boundary

$$\begin{gathered}\Phi(x) + \overline{\Phi(x)} + x\Phi'(x) + \Psi(x) = 0\\ \Phi(x + iH) + \overline{\Phi(x + iH)} + (x - iH)\,\Phi'(x + iH) + \Psi(x + iH) = 0\end{gathered} \tag{1.3}$$

Values of the functions on the left and right edges of the slit and marked by plus and

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minus superscripts, respectively, where the positive direction is selected from the point $y = a$ to $y = b$.

We will use a well-known method* (*Karpenko, L.N., On an approximate method of solving a singular integral equation and its application to a problem of plane elasticity theory for a domain with slots, Candidate Dissertation, Novosibirsk, 1965. Modifications of this method are described for example, in /8/.) to reduce the problem to a singular integral equation (SIE). Starting from the boundary conditions (1.2) and the symmetry of the problem, we introduce the following representation of the desired functions

$$\Phi(z) = \Phi_0(z) + \frac{1}{2\pi} I_1(-iz), \quad \Psi(z) = \Psi_0(z) - izI_2(-iz) \tag{1.4}$$

$$I_n(z) = \frac{1}{2\pi}\int_a^b \frac{\varphi(\eta)}{(\eta - z)^n}\,d\eta, \quad \varphi(\eta) = \frac{2\mu}{1+\varkappa}\frac{d}{d\eta}[u^+(\eta) - u^-(\eta)]$$

where the functions Φ_0, Ψ_0 are regular in a continuous strip, $u^\pm(\eta)$ are displacements of the slit edges in the direction of the Ox axis, and μ and $\varkappa$ are elastic constants /10/. Therefore, $\varphi(\eta)$ is a real function. The boundary condition (1.2) results in the relationship

$$2I_1(y) + \Phi_0(iy) + \overline{\Phi_0(iy)} + iy\Phi_0'(iy) - \Psi_0(iy) = p(y) \tag{1.5}$$

The boundary conditions (1.3) enable the functions $\Phi_0(z)$ and $\Psi_0(z)$ to be expressed in terms of the new unknown function $\varphi(\eta)$, after which (1.5) becomes a SIE in $\varphi(\eta)$ whose solution should also satisfy the condition of single-valuedness of the displacement

$$\int_a^b \varphi(\eta)\,d\eta = 0 \tag{1.6}$$

Therefore, we obtain the following boundary value problem for Φ_0 and Ψ_0:

$$\begin{gathered}\Phi_0(t) + \overline{\Phi_0(t)} + \bar{t}\Phi_0'(t) + \Psi_0(t) = -I_1(-it) - I_1(i\bar{t}) + \\ i(t+\bar{t})\,I_2(-it), \quad t = x, \quad t = x + iH, \quad -\infty < x < \infty\end{gathered} \tag{1.7}$$

2. We will use the Fourier transform

$$\Phi_0(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(\xi)\,e^{iz\xi}\,d\xi, \quad \Psi_0(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} B(\xi)\,e^{iz\xi}\,d\xi$$

to solve problem (1.7)

We find from the boundary conditions for (1.7)

$$A(\xi) = -\frac{1}{\sqrt{2\pi}}\int_a^b \varphi(\eta)\,\alpha(\eta,\xi)\,d\eta, \quad B(\xi) =$$

$$-\frac{1}{\sqrt{2\pi}}\int_a^b \varphi(\eta)\,\beta(\eta,\xi)\,d\eta$$

$$\begin{gathered}\alpha(\eta,\xi) = \{(1 - e^{-2H\xi})[(2\eta\xi - 1)e^{H\xi} + e^{-(H-2\eta)\xi}] + \\ 2H\xi e^{-H\xi}[(2H\xi - 2\eta\xi - 1)e^{2\eta\xi} + 1]\}\,\gamma(\eta,\xi), \quad \xi > 0\end{gathered}$$

$$\alpha(\eta,\xi) = -e^{H\xi}\alpha(H-\eta, -\xi), \quad \xi < 0$$

$$\beta(\eta,\xi) = -\alpha(\eta,-\xi) + \xi\frac{\partial}{\partial\xi}\alpha(\eta,\xi) + \begin{cases} e^{-\eta\xi}, & \xi > 0 \\ -(1 + 2\eta\xi)\,e^{\eta\xi}, & \xi < 0\end{cases}$$

$$\gamma(\eta,\xi) = e^{-(H+\eta)\xi}[(1 - e^{-2H\xi})^2 - 4H^2\xi^2 e^{-2H\xi}]^{-1}$$

Now we can write

$$\Phi_0(z) = \frac{1}{2\pi}\int_a^b L(z,\eta)\,\varphi(\eta)\,d\eta \tag{2.1}$$

$$z\Phi_0'(z) + \Psi_0(z) = \frac{1}{2\pi}\int_a^b N(z,\eta)\,\varphi(\eta)\,d\eta$$

$$L(z,\eta) = -\int_0^{\infty}[e^{iz\xi}\alpha(\eta,\xi) - e^{-(H-iz)\xi}\alpha(H-\eta,\xi)]\,d\xi \tag{2.2}$$

$$N(z,\eta)=\int\limits_0^\infty \{[(1+iz\xi-i\bar{z}\xi)e^{iz\xi}+e^{-iz\xi}]\,\alpha(\eta,\xi)+$$
$$[e^{iz\xi}+(1-i\bar{z}\xi)e^{-iz\xi}]\,\alpha(\eta,-\xi)-e^{(-\eta+iz)\xi}+(1-2\eta\xi)e^{-(\eta+iz)\xi}\}\,d\xi$$

Substituting relationships (2.1) into (1.5) we obtain the desired SIE. A number of authors /7-9/ obtained similar equations, nevertheless no one noted that the integrals (2.2) (they are represented in a different form in the papers of other authors) converge slowly, which does not enable acceptable accuracy to be achieved in practice in a number of cases. The slow convergence of these integrals is caused by the presence of components of the order of $e^{-(\eta+y)\xi}$, $e^{-(2H-\eta-y)\xi}$ etc., in the integrands. Consequently, it is advisable to extract the components mentioned and to integrate them in finite form, which will result in the following expressions:

$$L(z,\eta)=L_1(z,\eta)-L_1(iH-z,H-\eta),\ N(z,\eta)=\tag{2.3}$$
$$N_1(z,\eta)-N_1(iH-z,H-\eta)$$
$$L_1(z,\eta)=\frac{\eta+iz}{(\eta-iz)^2}+\int\limits_0^\infty\{f(\eta,\xi)e^{-(H+iz)\xi}+g(\eta,\xi)e^{-(H-iz)\xi}\}\,d\xi$$
$$N_1(z,\eta)=-\frac{4}{\eta-iz}+\frac{4i(z-\bar{z})\eta}{(\eta-iz)^3}+\frac{8\eta}{(\eta-iz)^2}-$$
$$\int\limits_0^\infty(\{f(\eta,\xi)[1-i(z-\bar{z})\xi]+g(\eta,\xi)\}e^{-(H+iz)\xi}+$$
$$\{f(\eta,\xi)+[1+i(z-\bar{z})\xi]\,g(\eta,\xi)\}e^{-(H-iz)\xi})\,d\xi$$
$$f(\eta,\xi)=[1-e^{-2H\xi}+2H\xi(2\eta\xi-1)]\,\gamma(\eta,\xi)$$
$$g(\eta,\xi)=[(1-2\eta\xi)(1-e^{-2H\xi}+4H^2\xi^2)-2H\xi]\,\gamma(\eta,\xi)$$

Substitution of the expressions obtained into (1.5) yields the following SIE:

$$\frac{1}{\pi}\int\limits_a^b\frac{\varphi(\eta)}{\eta-y}\,d\eta+\frac{1}{\pi}\int\limits_a^b K(y,\eta)\,\varphi(\eta)\,d\eta=p(y)\tag{2.4}$$
$$K(y,\eta)=M(y,\eta)-M(H-y,H-\eta)$$
$$M(y,\eta)=\frac{y^2+4y\eta-\eta^2}{(y+\eta)^3}+\frac{1}{2}\int\limits_0^\infty M_1(y,\eta,\xi)\,d\xi$$
$$M_1(y,\eta,\xi)=g(\eta,\xi)[(3-2y\xi)\,e^{-(H+y)\xi}+e^{-(H-y)\xi}]+$$
$$f(\eta,\xi)[e^{-(H+y)\xi}+(3+2y\xi)\,e^{-(H-y)\xi}]$$

The first component in the expression for $M(y,\eta)$ corresponds to a regular SIE kernel for the crack in a half-plane /11/, as might have been expected.

The condition of single-valuedness of the displacements (1.6) should be taken into account when solving (2.4).

3. Formulas (1.4), (2.1) and (2.3) enable the stress distributions to be calculated by means of the known function $\varphi(\eta)$:

$$\sigma_x=\operatorname{Re}\frac{1}{\pi}\int\limits_a^b\frac{2\eta+3iz+i\bar{z}}{2(\eta+iz)^2}\,\varphi(\eta)\,d\eta+\tag{3.1}$$
$$\frac{1}{\pi}\int\limits_a^b[E(z,\eta)-E(iH-z,H-\eta)]\,\varphi(\eta)\,d\eta$$
$$E(z,\eta)=-\operatorname{Re}\frac{\eta^2-2i\eta(z-\bar{z})+z^2}{(\eta-iz)^3}+\frac{1}{2}\int\limits_0^\infty M_1(y,\eta,\xi)\cos x\xi\,d\xi$$
$$\sigma_y=\operatorname{Re}\frac{1}{\pi}\int\limits_a^b\frac{2\eta+i(z-\bar{z})}{2(\eta+iz)^2}\,\varphi(\eta)\,d\eta+$$
$$\frac{1}{\pi}\int\limits_a^b[F(z,\eta)-E(iH-z,H-\eta)]\,\varphi(\eta)\,d\eta$$
$$F(z,\eta)=\operatorname{Re}\frac{3\eta^2-2i\eta(z-\bar{z})+3z^2}{(\eta-iz)^3}+\frac{1}{2}\int\limits_0^\infty M_{2-}(y,\eta,\xi)\cos x\xi\,d\xi$$
$$\tau_{xy}=-\operatorname{Re}\frac{x}{\pi}\int\limits_a^b\frac{\varphi(\eta)}{(\eta+iz)^2}\,d\eta+\frac{1}{\pi}\int\limits_a^b[G(z,\eta)-G(iH-z,H-\eta)]\,\varphi(\eta)\,d\eta$$

$$G(z, \eta) = 2\,\mathrm{Im}\frac{\eta^2 - i\eta(z-\bar z) + z^2}{(\eta - iz)^3} + \frac{1}{2}\int_0^\infty M_{2+}(y, \eta, \xi)\sin x\xi\, d\xi$$
$$M_{2\pm}(y, \eta, \xi) = f(\eta, \xi)[(1 \pm 2y\xi)\, e^{-(H-y)\xi} - e^{-(H+y)\xi}] \pm$$
$$g(\eta, \xi)[e^{-(H-y)\xi} - (1 \mp 2y\xi)\, e^{-(H+y)\xi}]$$

4. We will consider the elastic equilibrium of a strip with a central crack under tension by a constant stress $\sigma_y = p_0$ applied at infinity. In this case the right-hand side of (2.4) is $p(y) = -p_0$ while the upper limit of integration is $b = H - a$. Let $(H - 2a)/2 = l$, where $2l$ is the crack length, and $\lambda = (H - 2a)/H = 2l/H$. Making the change of variable

$$H\xi = \sigma,\ \eta = l\tau + H/2,\ y = l\tau_0 + H/2,\ -1 < \tau,\ \tau_0 < 1 \tag{4.1}$$

in (2.4), we transfer to integration in the segment $[-1,1]$

$$\frac{1}{\pi i}\int_{-1}^{1}\frac{\varphi(\tau)}{\tau - \tau_0}d\tau + \frac{1}{\pi i}\int_{-1}^{1}K_1(\tau_0, \tau)\,\varphi(\tau)\,d\tau = -p_0 \tag{4.2}$$
$$\int_{-1}^{1}\varphi(\tau)\,d\tau = 0,\quad K_1(\tau_0, \tau) = lK(y, \eta)$$

We find the kernel $K_1(\tau_0, \tau)$ numerically by using the Gauss-Laguerre quadrature formula /12/ with 15 nodes. As a numerical experiment showed, the error here did not exceed 0.001%.

We represent the unknown function $\varphi(\tau)$ in the form

$$\varphi(\tau) = -p_0 u(\tau)/\sqrt{1 - \tau^2} \tag{4.3}$$

($u(\tau)$ is the new unknown function). Using (4.3) we apply a Gauss-Chebyshev type quadrature formula /13/ to (4.2). We consequently obtain a system of linear algebraic equations in $u_i \approx u(\tau_i)$

$$\frac{1}{n}\sum_{i=1}^{n}u_i\left[\frac{1}{\tau_i - \tau_{0k}} + K_1(\tau_{0k}, \tau_i)\right] = 1 \quad (k = 1, 2, \ldots, n-1) \tag{4.4}$$
$$\sum_{i=1}^{n}u_i = 0$$
$$\tau_i = \cos\frac{2i-1}{2n}\pi \quad (i = 1, 2, \ldots, n)$$
$$\tau_{0k} = \cos\frac{\pi k}{n} \quad (k = 1, 2, \ldots, n-1)$$

The stress intensity factor (SIF) at the tip of a central recilinear crack in a strip is calculated as follows by using an interpolation formula:

$$k_1/(p_0\sqrt{\pi l}) = -u(1) \tag{4.5}$$
$$u(1) = -\frac{1}{n}\sum_{i=1}^{n}(-1)^n\sqrt{\frac{1+\tau_i}{1-\tau_i}}\,u_i$$

where u_i is the solution of system (4.4). The results of computing the dimensionless SIF at the tip of a central crack in a strip stretched by a constant normal stress at infinity by means of (4.5) are presented below:

λ	0.1	0.3	0.5	0.7	0.9	0.95
$k_1/(p_0\sqrt{\pi l})$	1.0060	1.0577	1.1867	1.4882	2.5796	3.667

The error in the calculations, not exceeding 0.1% in all cases, was checked by comparing the results of calculations obtained for different accuracy of the algebraic approximation of the integral Eq.(4.2), which is determined by the order n of the corresponding system of linear algebraic Eqs.(4.4).

To 0.007% accuracy the results for $\lambda \leqslant 0.7$ agree with the data in /4, 7/. The difference from the results represented in /3, 8/ reaches 0.8% and 0.6%, respectively in the same range of λ. The error check performed for the calculations enables us to consider the results obtained as most accurate.

The problem of an eccentrically located inner crack can be examined in the same way as above.

5. We consider the elastic equilibrium of a strip with an edge crack for two kinds of loads of infinity: tension by a constant stress $\sigma_y = p_{01}$ or bending by the moment M. In this

case the right-hand side of (2.4) is $p(y) = -p_{01}$ or $p(y) = -p_{02}(1 - 2y/H)$, where $p_{02} = 6M/(dH^2)$, and d is the plate thickness which we set equal to one. Let $a = 0$ and $b = l$, where l is the crack length, and $\lambda = l/H$. Making the change of variable

$$H\xi = \sigma,\ \eta = l\tau,\ y = l\tau_0,\ -1 < \tau,\ \tau_0 < 1 \tag{5.1}$$

in (2.4), we transfer to integration in the segment [0,1]. We consequently obtain relationships analogous to (4.2) for the appropriate change in the limits of integration and the replacement of $-p_0$ by $p(\tau_0)$.

The kernel $K_1(\tau_0, \tau)$ will be evaluated exactly as in Sect.4.

We solve the SIE obtained by a numerical method /14/ which is especially effective in the examination of intersecting cracks or those emerging on the body boundary. To this end we will represent the unknown function $\varphi(\tau)$ in the form

$$\varphi(\tau) = -p_{0r}\sqrt{\frac{\tau}{1-\tau}}\,u_r(\tau) \tag{5.2}$$

where $u_r(\tau)$ are new unknown functions ($r = 1$ for tension and $r = 2$ for bending). Although the factor $\sqrt{\tau/(1-\tau)}$ in representation (5.2) does not fully describe the behaviour of the desired function at the point $\tau = 0$, however as is shown in /14/, its utilization enables a numerical solution of the SIE of similar problems to be obtained effectively.

We apply a quadrature formula of the semi-open type /14/. We consequently obtain a system of linear algebraic equations in $u_{rj} \approx u_r(\tau_j)$

$$\sum_{j=1}^{n} A_j u_{rj}\left[\frac{1}{\tau_j - \tau_{0k}} + K_1(\tau_{0k}, \tau_j)\right] = \begin{cases} 1, & r = 1 \\ 1 - 2\lambda\tau_{0k}, & r = 2 \end{cases} \tag{5.3}$$

$$A_j = \frac{1}{n}\sin^2\frac{j\pi}{2n} \quad (j = 1, 2, \ldots, n-1), \quad A_n = \frac{1}{2n}$$

$$\tau_j = \sin^2\frac{j\pi}{2n} \quad (j = 1, 2, \ldots, n), \quad \tau_{0k} = \sin^2\frac{2k-1}{4n}\pi$$

$$(k = 1, 2, \ldots, n)$$

The SIF at the apex of a rectilinear edge crack located in a strip is calculated as follows:

$$k_{1r}/(p_{0r}\sqrt{\pi l}) = -\sqrt{2}\,u_r(1) \quad (r = 1, 2) \tag{5.4}$$

The values $u_r(1) \approx u_{rn}$ for utilization of the method mentioned are determined directly from the solution of the system of linear algebraic equations (5.3) and not by using an interpolation formula of the type of the second formula in (4.5) which is an additional source of error. This is one of the advantages of this method.

The table shows results of computing the dimensionless SIF at the apex of an edge in a strip subjected to a constant tensile stress p_{01} (the upper part of the table) or a bending moment M (the lower part), respectively by means of (5.4) and also the results of other authors. The error in the calculation was checked exactly as in the case of the central crack. All the calculations were performed with double precision on an ES series electronic computer. All the numbers presented are valid.

Table

Source	$\lambda = 0{,}05$	0,1	0,3	0,5	0,7	0,9	0,95
[5]	1,14	1,19	1,67	2,83	6,38	34,6	99,3
[6]	1,140	1,189	1,660	2,826			
[8]		1,189	1,659	2,820	6,340	33,92	90,66
[9]	1,14	1,19	1,66	2,83	6,38	34,8	102
(5.4)	1,1399	1,1892	1,6599	2,825	6,36	34,6	99,4
[5]	1,07	1,04	1,11	1,48	2,72	12,5	
[8]		1,047	1,123	1,494	2,717	12,19	31,26
[9]	1,07	1,04	1,13	1,50	2,73	12,4	
(5.4)	1,0709	1,0472	1,1242	1,4973	2,726	12,5	34,4

It is seen that the discrepancy in the results for an edge crack obtained by different authors reaches 10%. The results obtained here are in good agreement with the data in /9/. The check performed for the calculation error enables us to regard the results obtained as most accurate.

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DIFFUSION CHARGING OF PARTICLES IN ONE-DIMENSIONAL WEAKLY IONIZED AEROSOL FLOWS*

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Electrohydrodynamics is used in /1, 2/ to study one-dimensional flows of aerosol particles carrying a bipolar charge in electric field, in the case when the parameters of the electrohydrodynamic (EHD) interaction between the phases are small. It is assumed that the radius of the aerosol particles is small and that the charging process is governed by the thermal motion of the ions towards their surface. The case of large Peclet numbers is considered, the numbers constructed in accordance with the characteristic dimension of the problem, i.e. by neglecting the contribution of diffusion towards the total macroscopic flows of the ions. The reaction rate at which the ions transfer their charge to the particles, is assumed to be finite. A digital computer is used to study the dependence of the flow parameters on the reaction rate constant and the particle density. The results of the calculations are compared with the analytic solution of the problem obtained for low-concentration aerosols in the case of large electrical Reynolds numbers.

EHD flows of weakly ionized aerosols with volume ion sources occur in various natural and technological processes caused, for example, by external radioactivity /1-3/. In such flows the particles of the disperse phase can become charged as a result of precipitation of ions of predominantly one sign. In order to study the special features of the interphase charge transfer in weakly ionized aerosols, in the presence of volume ionization, it is best to study

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